

CONSTRUCTION OF THE DUAL GINZBURG-LANDAU THEORY FROM THE LATTICE QCD

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We briefly review the QCD physics and then introduce recent topics on the confinement physics. In the maximally abelian (MA) gauge, the low-energy QCD is abelianized owing to the effective off-diagonal gluon mass $M_{\text{off}} \simeq 1.2\text{GeV}$ induced by the MA gauge fixing. We demonstrate the construction of the dual Ginzburg-Landau (DGL) theory from the low-energy QCD in the MA gauge in terms of the lattice QCD evidences on infrared abelian dominance and infrared monopole condensation.

1. What is the Quark Confinement Physics ?

About two centuries ago, Dalton and Avogadro introduced the idea of “atoms” and “molecules” into the modern science. Since then, to find out the elementary object becomes one of the most important subject in the modern science.

According to the experimental progress, the “elementary object” has been changed. The atom is divided into a nucleus and electrons, and the nucleus is divided into nucleons... Then, to separate out the more fundamental object becomes one of the most central philosophy in the particle physics.

Nowadays, the nucleon is recognized to consist of quarks and gluons. Nevertheless, the individual quark cannot be separated from the nucleon, but is confined inside the nucleon. We can never observe the raw quark directly, and quarks are strongly bound each other. Then, the Quark Physics is a sort of many-body physics. This is the most different point from the other subjects in the elementary particle physics.

Furthermore, such a curious quark feature is deeply related to the property of the QCD vacuum, which is not a large “empty box” but is regarded as the “condensed matter” with a finite values of the gluon condensate and the quark condensate. Therefore, the Quark Physics requires the total sense

of the particle physics, many-body physics and condensed matter physics. (So, the Quark Physics looks like a jigsaw puzzle. We need individual precise knowledge keeping the total balance.)

2. What is the Strategy of the Quark Physics ?

To seek the elementary law is one of the most important aims in the particle physics. However, the aim of the Quark Physics is quite different, because we already know the elementary law of QCD. Surely, QCD tells us the “rule” of the elementary interaction between quarks and gluons, but to solve QCD is another difficult problem.

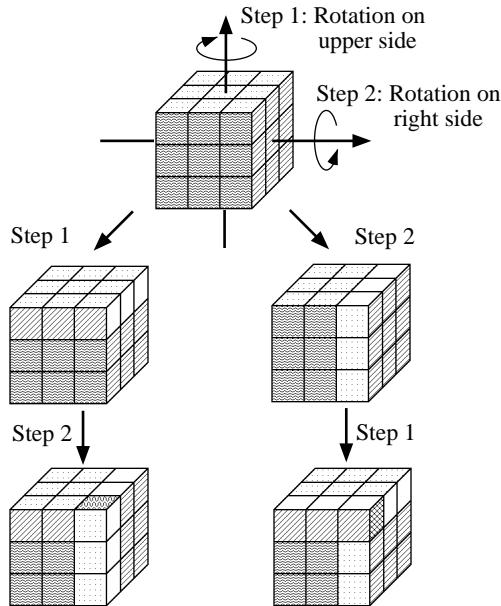


Figure 1. The difficulty and the interest of the Rubik cube originate from non-commutable operations based on the nonabelian nature of the rotational group. The configuration after Step 1 and Step 2 depends on the order of these operations. Can you find a nonabelian nature of QCD in the Rubik cube ?

Then, the QCD physic is similar to the Rubik Cube ! The rule is simple, but to solve is rather difficult. This difficulty is also based on the non-commutable procedures of the “nonabelian” rotational process as shown in Fig.1. Here, a kind of “locality” of the rotational process leads to variety and extremely large number of configurations, which makes this

puzzle difficult and interesting. Thus, one may be able to find interesting analogy between the Rubik cube and QCD, which is characterized by the nonabelian local symmetry.

The present strategy for the Quark Physics seems to be the both-side attack from QCD and hadron phenomena. This is similar to the Nuclear Physics strategy. In fact, we already know the experimental data of the nuclear force between nucleons, which is the elementary law of nuclear dynamics. Nevertheless, we need several phenomenological models such as the liquid drop model, the shell model, the cluster model, the interacting boson model to understand the many aspects of the nuclear properties. (No nuclear physicist would say that nuclear physics is to solve the Schrödinger equation of nucleons.) Similarly in the nuclear physics, the linkage of the “both sides”, QCD and hadron phenomena, is one of the most important aims in the Quark Physics.

3. Why is the QCD Physics difficult ?

The main difficulty of QCD originates from the nonabelian nature and the strong-coupling nature in the infrared region below 1 GeV. In the ultra-violet region, the QCD coupling becomes weak and then we can use the perturbation technique, where the nonabelian part can be treated as the perturbative interaction. But, in the infrared region, the nonabelian and the strong-coupling nature becomes significant.

To see the difficulty on the nonabelian property of QCD, let us consider the simple electro-magnetic system. In the ordinary electro-magnetism, we can individually consider the partial electro-magnetic field formed by each charge, and the total electro-magnetic field can be obtained by adding these individual solutions. Here, additivity of the solution plays the key role, and this additivity originates from the linearity of the field equation, $\partial_\mu F^{\mu\nu} = j^\nu$, in the electro-magnetism.

On the other hand, owing to the nonabelian nature, the QCD field equation becomes nonlinear as

$$\partial_\mu G^{\mu\nu} + ig[A_\mu, G^{\mu\nu}] = j^\nu, \quad (1)$$

and then it is difficult to solve it even at the classical level, because the solution does not hold the additivity. So, we cannot divide the color-electromagnetic field into each part formed by each quark. Instead, the QCD system is to be analyzed as a whole system. (Furthermore, the QCD vacuum is the nontrivial medium with the gluon condensate.) For the analysis of QCD, the nonabelian nature provides a serious difficulty.

4. Abelianization of QCD - Infrared Abelian Dominance in the Maximally Abelian Gauge

The nonabelian nature is one of the characteristic features of QCD. However, by taking the maximally abelian (MA) gauge in QCD, one can make the nonabelian (off-diagonal) ingredients of QCD inactive for the infrared QCD properties such as quark confinement and chiral-symmetry breaking. We call these phenomena as the infrared abelian dominance in the MA gauge.¹⁻¹⁵

In the Euclidean QCD, the MA gauge is defined so as to minimize the total amount of off-diagonal gluons, $R_{\text{off}} \equiv \int d^4x \sum_{\mu,\alpha} |A_\mu^\alpha(x)|^2$, by the $SU(N_c)$ gauge transformation. Here, $A_\mu^\alpha(x)$ denotes the off-diagonal gluon in the Cartan decomposition, $A_\mu = \vec{A}_\mu(x) \cdot \vec{H} + A_\mu^\alpha E^\alpha$. In the MA gauge, by removing the off-diagonal gluons, QCD can be well approximated as an abelian gauge theory like the electro-magnetism keeping the essence of the infrared QCD properties. This approximation is called as abelian projection.

Owing to this remarkable feature of MA gauge fixing, the gluon field can be approximated to be abelian as $A_\mu^a(x)T^a \simeq \vec{A}_\mu(x) \cdot \vec{H}$ for the argument on long-distance physics. Accordingly, the field equation of the abelian-projected QCD becomes linear like the Maxwell equation,

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = k^\nu, \quad (2)$$

with the color-electric current j^μ and the color-magnetic current k^μ . Thus, the additivity on color-electromagnetic fields $F^{\mu\nu}$ works in the abelian-projected QCD in the MA gauge. This is the most attractive point of the MA gauge.

5. Origin of Infrared Abelian Dominance - Mass Generation of Off-diagonal Gluons in the MA Gauge

QCD in the MA gauge exhibits the following interesting properties.

- In the MA gauge, the nonabelian $SU(N_c)$ gauge symmetry is reduced into the abelian $U(1)^{N_c-1}$ gauge symmetry with the global Weyl symmetry.¹¹
- In the MA gauge, the off-diagonal gluon amplitude is strongly suppressed. As the result, there appears a strong randomness of the off-diagonal gluon phase in the MA gauge, which is found to be mathematical origin of abelian dominance for confinement.^{10,12}

- As a remarkable fact, the color-magnetic monopole appears as the topological object in the MA gauge.^{2,3,7,10,12}
- Infrared abelian dominance holds for quark confinement and dynamical chiral-symmetry breaking in the MA gauge.

As the physical origin of infrared abelian dominance, we recently find out effective mass generation of off-diagonal gluons in the MA gauge using the lattice QCD.^{10,13} The effective off-diagonal gluons mass is measured as $M_{\text{off}} \simeq 1.2$ GeV from the behavior of the off-diagonal gluon propagator $G_{\mu\nu}^{\text{off}}$ in the SU(2) lattice QCD as shown in Fig.2.

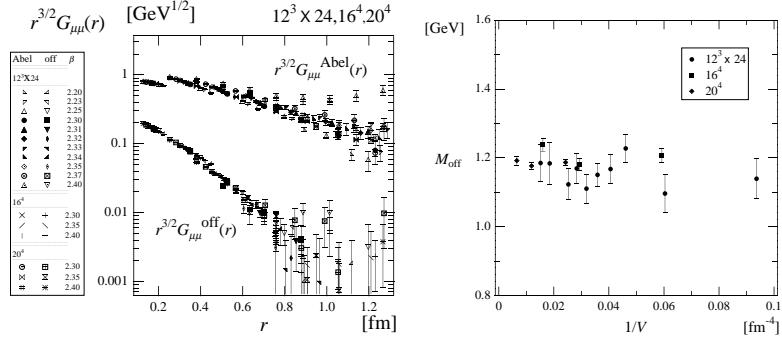


Figure 2. (a) The logarithmic plot of $r^{3/2}G_{\mu\mu}^{\text{off}}(r)$ and $r^{3/2}G_{\mu\mu}^{\text{Abel}}(r)$. The slope corresponds to the effective mass. (b) The off-diagonal gluon mass M_{off} v.s. inverse lattice volume.

In the MA gauge, the off-diagonal gluon behaves as a massive field with a large mass about 1.2 GeV and then becomes inactive for the long-distance physics, similar to the negligible contribution of the massive weak boson to the long-distance force in the Weinberg-Salam model. This is the essence of the infrared abelian dominance in the MA gauge.

6. Appearance of Color-Magnetic Monopoles in MA gauge in QCD

In the MA gauge, the nonabelian $SU(N_c)$ gauge symmetry is reduced into the abelian $U(1)^{N_c-1}$ gauge symmetry. Owing to this partial gauge fixing, the color-magnetic monopole appears as the topological object corresponding to the nontrivial homotopy group, $\Pi_2(SU(N_c)/U(1)^{N_c-1}) = Z^{N_c-1}$, as was first pointed out by 't Hooft in 1980.²

You may suspect the reality of the monopole in QCD. Then, do you know the following fact ? If the Weinberg-Salam model had simpler gauge symmetry $SU(2)_L$ instead of $SU(2)_L \times U(1)$, this theory predicted the existence of the magnetic monopole with the mass about 100 GeV ! So, the experimentalists might make much effort to create and to observe the magnetic monopole in order to get the Nobel prize !! Unfortunately (?), the symmetry of the Weinberg-Salam model is $SU(2)_L \times U(1)$, and therefore this theory predicted the neutral current instead of magnetic monopoles.

Grand Unified Theory (GUT) actually predicts the quite heavy magnetic monopole (GUT monopole) with the mass about 10^{16} GeV. The appearance of color-magnetic monopoles in the MA gauge in QCD has similar topological origin to the GUT monopole. Actually in the lattice QCD simulation in the MA gauge, there appears a global network of the monopole world-line covering the whole system as shown in Fig.3(a).^{10,15}

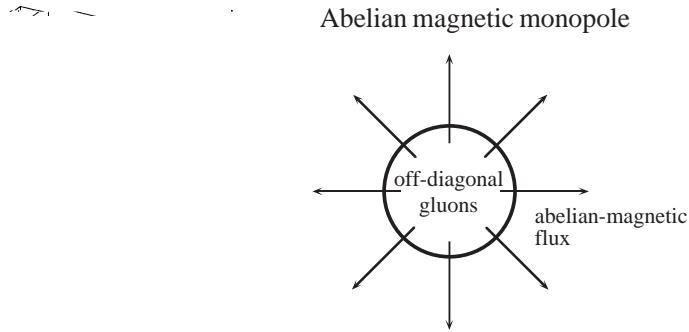


Figure 3. (a) The \mathbf{R}^3 projected world-line of the color-magnetic monopole in the MA gauge in the $SU(2)$ lattice QCD with $16^3 \times 4$ and $\beta=2.2$ (confinement phase). (b) The structure of the color-magnetic monopole. There remains large off-diagonal gluon component near the monopole center.

Here, even in the MA gauge, where the off-diagonal gluon element is strongly suppressed, around monopoles, there remains large off-diagonal gluon component. This off-diagonal-gluon-rich region around the monopole provides an “intrinsic size” and the structure of the monopole as shown in Fig.3(b), like the 't Hooft-Polyakov monopole.^{10,12,15} (The instanton is also an important topological object appearing in the nonabelian gauge manifold, and obviously the instanton needs full $SU(2)$ components for existence. Therefore, instantons tend to appear in the off-diagonal-gluon-rich region around the monopole world-line in the MA gauge, which predicts the local

correlation among monopoles, instantons and off-diagonal gluons.^{10,15)}

Using the lattice QCD, we find that the dual gluon B_μ is massive in the MA gauge, which is an numerical evidence of monopole condensation and the dual Higgs mechanism.^{10,15} Then, quark confinement seems to be interpreted with monopole condensation, which was first proposed by Nambu in 1974.¹ Here, a color-singlet monopole appears as the dual Higgs particle,¹¹ and its fluctuation can be observed as a scalar glueball with $J^{CP} = 0^{++}$.

In fact, the dual superconductor scenario for quark confinement predicts a massive scalar glueball, like the Higgs particle in the Weinberg-Salam model.

7. Summary and Outlook

To conclude, the lattice QCD in the MA gauge exhibits *infrared abelian dominance* and *infrared monopole condensation*, and therefore the dual Ginzburg-Landau (DGL) theory³⁻¹⁵ can be constructed as the infrared effective theory directly based on QCD in the MA gauge.

Historically, an interesting interpretation of phenomena has eventually lead to new finding on the particle as follows.

- To interpret the origin of strong nuclear force, Yukawa predicted existence of mesons.
- Nambu and Goldstone proved that spontaneous chiral-symmetry breaking leads to inevitable existence of light pseudoscalar particles (pions).
- From the classification of hadrons, Gell-Mann and Zweig found quarks.
- In near future, from further studies on the quark-confinement mechanism, new important finding may be done on the existence or the feature of the monopole (a scalar glueball).

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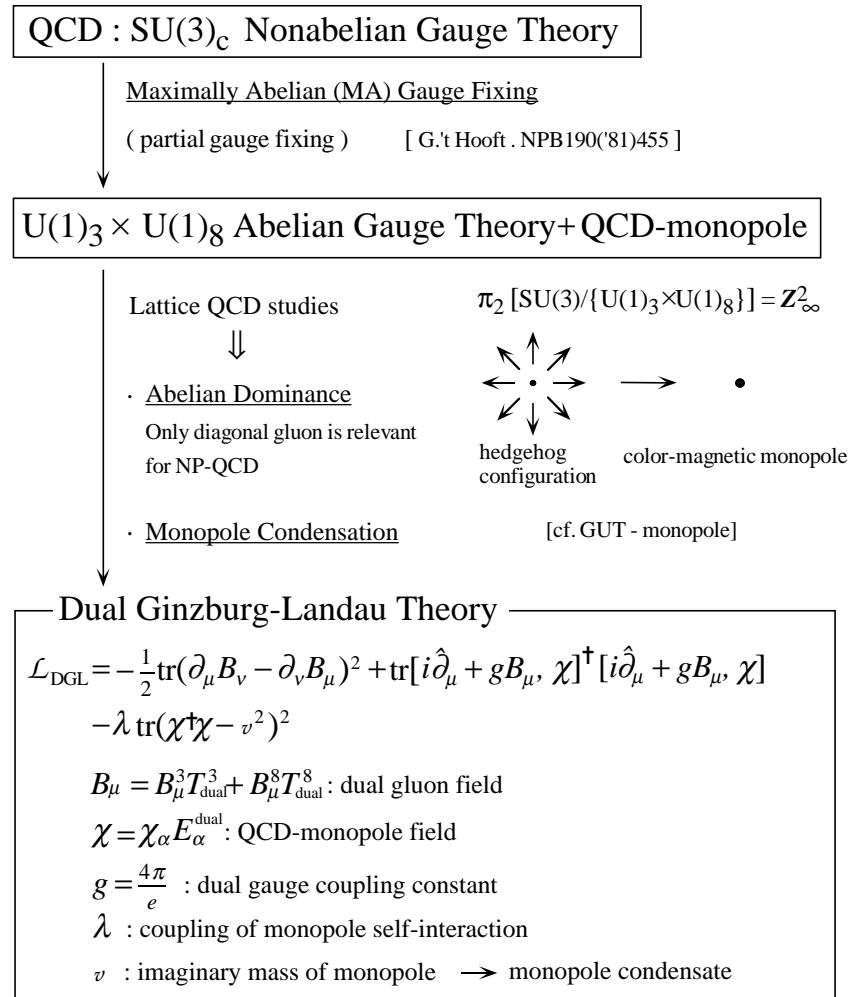
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From Lattice QCD to Dual Ginzburg-Landau Theory

— Infrared Effective Theory directly based on QCD —



Dual Gauge Symmetry is spontaneously broken instead of Gauge Symmetry

Figure 4. Construction of the dual Ginzburg-Landau (DGL) theory from the lattice QCD in the maximally abelian (MA) gauge.